

Lecture 8

gm/ID Sizing Method

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Preface

- This lecture was based on the following paper:
 - F. Silveira, D. Flandre, and P. G. A. Jespers, “A **gm/ID** based methodology for the design of CMOS analog circuits and its application to the **synthesis of a silicon-on-insulator** micropower OTA,” *IEEE J. Solid-State Circuits*, vol. 31, pp. 1314–1319, **Sep. 1996**.

Outline

- **Intrinsic gain stage**
- **gm/ID in different regions**
- **gm/ID versus normalized current ($I_D/(W/L)$)**
- **gm/ID sizing procedure**
- **Experimental result**
- **SOI technology for low-power circuits**
- **Summary**

Tradeoff btw Power & Speed

- CMOS analog circuits traditionally work in strong inversion (saturation)
- **Weak inversion region** → minimum power consumption; but slow
- Moderate inversion → good compromise in power and speed (future design interest)
- Design challenges:
 - Requiring **both** low power and high speed

Traditional Design Methodology

- Traditional optimization approaches → **SPICE** plus **numerical optimization** software
 - Disadvantage: lack of design insights
- Main stream methods emphasize “strong inversion”;
- Micropower design techniques exploit known “**weak inversion**” models.
- Symbolic or simple hand-calculation methods → better insights,
 - But lack simple and accurate hand models for moderate inversion

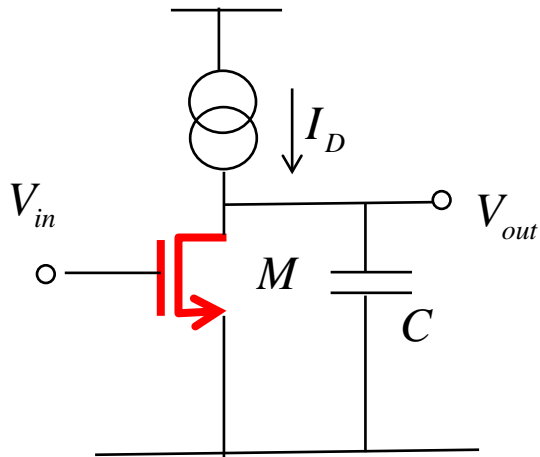
The g_m/I_D Methodology

- One single model that works in all operation regions.
- Focused on g_m/I_D ratio versus the normalized current $I_D/(W/L)$
 - the normalized current $I_D/(W/L)$ is characterized experimentally
 - or fitted with simple analytical models
- Helps design in moderate inversion for low-power circuits
 - Offering good compromise between speed and power.;
 - power lower but speed not bad!

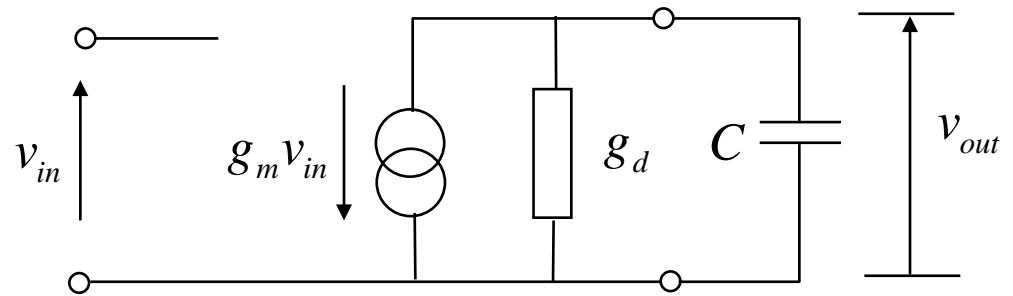
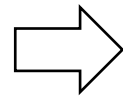
Motivation of g_m/I_D

- g_m/I_D is a **measure of the efficiency to translate current (i.e., power) into g_m (i.e., gain)**.
 - The greater g_m/I_D , the greater g_m is for a fixed I_D .
 - g_m/I_D is interpreted as a measure of the “ **g_m enhancement efficiency**”.
- It is **strongly** related to the performance of analog circuits.
- It also gives an indication of the device operating region.
- It can be used for transistor **sizing**.

Intrinsic Gain Stage (I.G.S.)



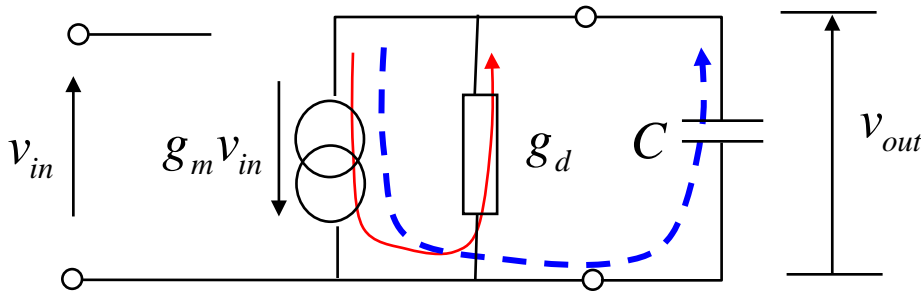
The common source transistor M is in saturation



The equivalent small-signal circuit

GBW

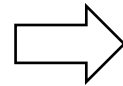
Gain-Bandwidth Product (GBW)



$$V_{out}(s) = H(s)V_{in}(s)$$

DC gain:

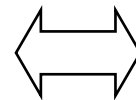
$$g_m v_{in} = -g_d v_{out}$$



$$H_{DC} = H(0) = \frac{v_{out}}{v_{in}} = -\frac{g_m}{g_d}$$

High freq gain:

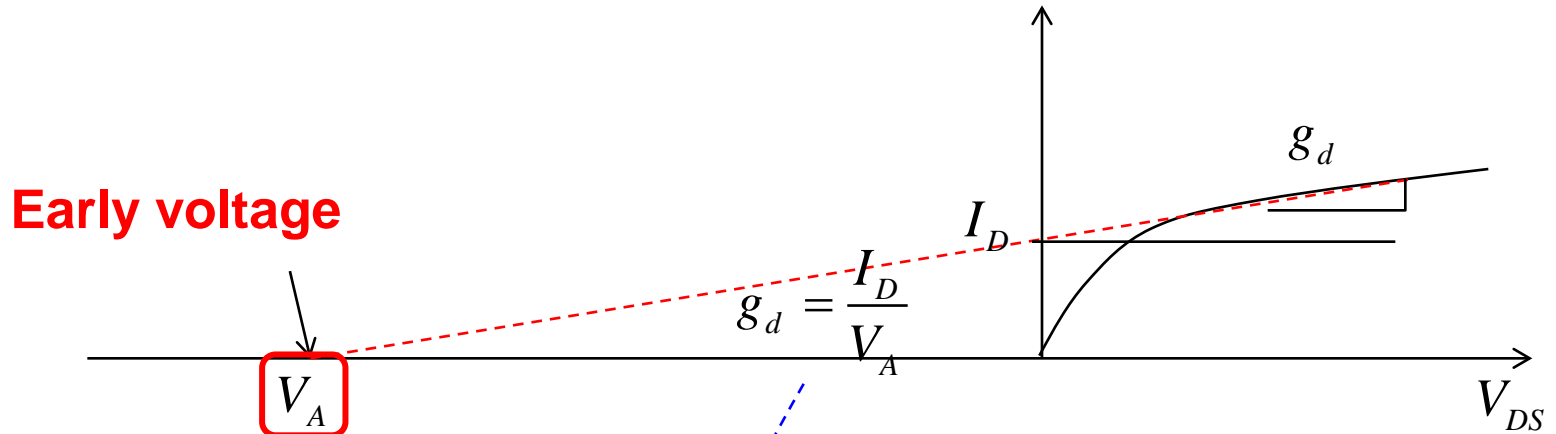
$$g_m v_{in} = -j\omega C v_{out}$$



$$H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{g_m}{j\omega C}$$

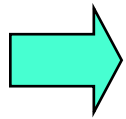
- At high frequencies, most of the current flows by the capacitor C.

Channel Length Modulation



$$\frac{v_{out}}{v_{in}} = -\frac{g_m}{g_d}$$

The Early voltage V_A controls the transistor small-signal output conductance, $g_d = I_D/V_A$.



$$A_{DC} = H(0) = \frac{v_{out}}{v_{in}} = -\frac{g_m}{g_d} = -\frac{g_m}{I_D} V_A$$

GBW (cont'd)

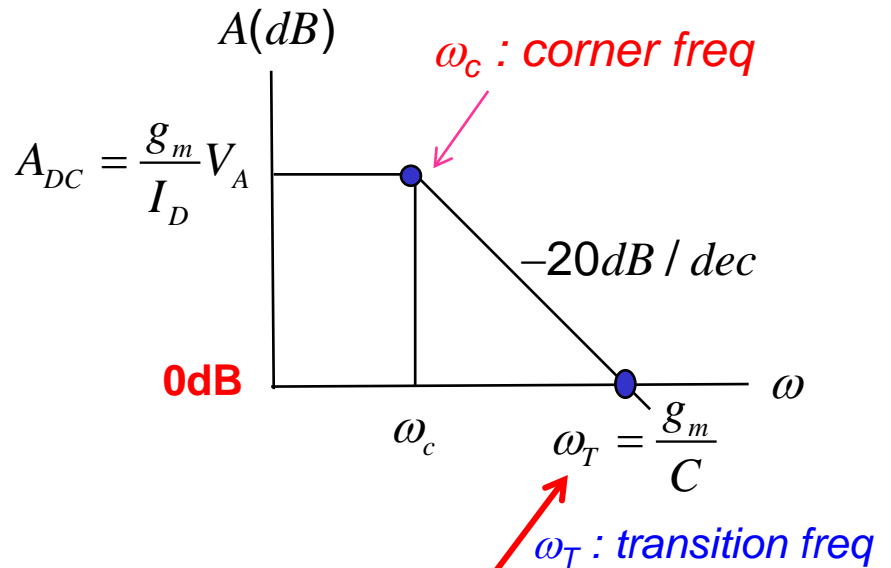
$$\left\{ \begin{array}{l} H(0) = -\frac{g_m V_A}{I_D} \\ H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{g_m}{j\omega C} \end{array} \right.$$

at corner freq

$$A_{DC} = |H(s)|_{s=0} = |H(s)|_{s=j\omega_c}$$

$$\frac{g_m V_A}{I_D} = \frac{g_m}{\omega_c C}$$

$$\frac{V_A}{I_D} = \frac{1}{\omega_c C} \quad \omega_c = \frac{I_D}{C V_A}$$



$$\omega_T = \omega_c A_{DC} = \frac{g_m}{C}$$

GBW

Calculation of gm/ID

The expression for gm/ID is derived as follows:

$$\frac{g_m}{I_D} = \frac{1}{I_D} \frac{\partial I_D}{\partial V_G} = \frac{\partial \ln I_D}{\partial V_G} = \frac{\partial \ln \left\{ \frac{I_D}{\left(\frac{W}{L}\right)} \right\}}{\partial V_G}$$

Called **normalized drain current**

$$I_{\square} \equiv \frac{I_D}{\left(\frac{W}{L}\right)}$$

ID normalized by the transistor size.

The derivative is **maximum** in the **weak inversion (WI)** region where the dependence of I_D versus V_G is exponential.

$$I_D = I_0 \exp\left(\frac{V_G}{nU_T}\right) \qquad \ln\left(\frac{I_D}{I_0}\right) = \frac{V_G}{nU_T}$$

Quadratic MOS Models

The connections between I_D , the W/L ratio, and g_m can be derived from the transistor large-signal model.

The classical MOS model is defined for the three regions:

1) **Strong inversion region (Quadratic Model):**

- When $V_{ov} = (V_G - V_{th}) > 0.2V$.

2) **Moderate inversion region;**

3) **Weak inversion region:** Once the current approaches $I_{D,min}$ (see the lecture on EKV), W/L must be increased fastly to further increase the DC gain.

Quadratic model in Strong Inversion

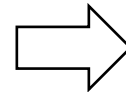
The quadratic expression of I_D for a MOS transistor in saturation:

$$I_D = \beta \frac{(V_G - V_{th})^2}{2n}$$

n is the slope factor ≈ 1

$$\beta = \mu C_{ox} \frac{W}{L}$$

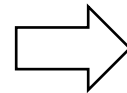
$$g_m = \frac{\partial I_D}{\partial V_G} = \beta \frac{V_G - V_{th}}{n} = \sqrt{\frac{2\beta I_D}{n}}$$



$$\frac{W}{L} = \frac{ng_m^2}{2\mu C_{ox}} \cdot \frac{1}{I_D}$$

We also have

$$g_m = \sqrt{\frac{2\beta I_D}{n}} = \frac{\beta}{n} (V_G - V_{th})$$



$$\frac{g_m}{I_D} = \frac{2}{(V_G - V_{th})}$$

Indep. of (W/L)

I_D is proportional to β (hence W), so is g_m .

(cont'd)

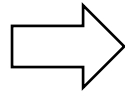
$$\boxed{\frac{W}{L} = \frac{ng_m^2}{2\mu C_{ox}} \cdot \frac{1}{I_D}} \quad \Rightarrow \quad \frac{\frac{W}{L}}{I_D} = \frac{n}{2\mu C_{ox}} \cdot \frac{g_m^2}{I_D^2} = \frac{n}{2\mu C_{ox}} \cdot \left(\frac{g_m}{I_D}\right)^2$$

$$\Rightarrow \quad \frac{g_m}{I_D} = \sqrt{\frac{2\mu C_{ox}}{n}} \frac{1}{\sqrt{I_D/W/L}}$$

gm/ID is inversely proportional to the sqrt of the normalized ID.

Relation in strong inversion

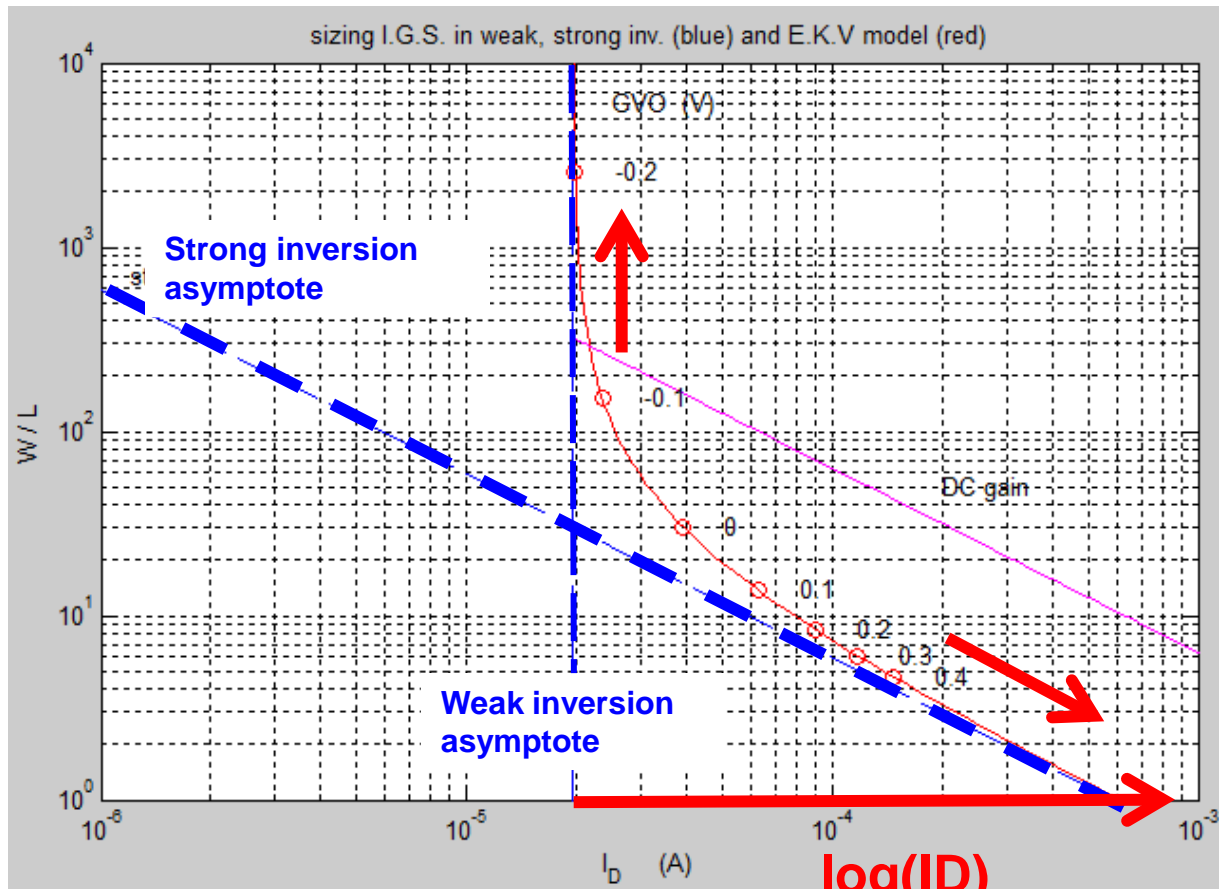
$$\frac{W}{L} = \frac{ng_m^2}{2\mu C_{ox}} \cdot \frac{1}{I_D}$$



$$\log \frac{W}{L} = \left(\log \frac{ng_m^2}{2\mu C_{ox}} \right) - \log I_D$$

Approx const in strong inversion

$\log(W/L)$



$I_D \rightarrow \text{small}$

$$A_{DC} = \frac{g_m V_A}{I_D}$$

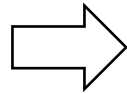
(derived for the
Intrinsic Gain Stage)

- It seems that the DC gain would increase to infinite as the drain current goes to 0.
- However, **as the current diminishes**, the transistor enters moderate and weak inversion, where the quadratic model for the drain current fails.

Weak Inversion

The drain current in weak inversion is given by the exponential I-V relation:

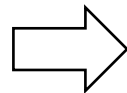
$$I_D = I_0 \exp\left(\frac{V_G}{nU_T}\right)$$



$$g_m = \frac{\partial I_D}{\partial V_G} = \frac{I_D}{nU_T}$$



i.e., gm/ID is approx. const in weak inversion.



$$I_{D,\min} = nU_T g_m$$

where n is the **subthreshold slope factor** and U_T the **thermal voltage**.

In weak inversion, the **drain current ID alone determines gm**, which in turn determines the **GBW**.

$$GBW = 2\pi f_T = \frac{g_m}{C} \quad (\text{for I.G.S.})$$

Moderate Inversion Region

The candidate model for **moderate inversion** is (see Jespers 2010, Chapter 4):

$$\frac{W}{L} = \frac{ng_m^2}{2\mu C_{ox}} \frac{1}{(I_D - I_{D,\min})}$$

The expression is valid in all regions, from strong to weak inversion.

Exercise: Design an I.G.S. with

- load $C = 1$ pF;
- transition frequency $f_T = 100$ MHz;
- $\mu C_{ox} = 4 \times 10^{-4}$ AV²;
- **slope factor** $n = 1.2$;
- Early voltage $V_A = 10$ V .

Figure

Plot of aspect ratio W/L vs I_D of an (ideal) Intrinsic Gain Stage.
 The numbers besides the **RED** circles show the **Overdrive Gate Voltage**
 $V_{ov} = (V_G - V_{th})$.

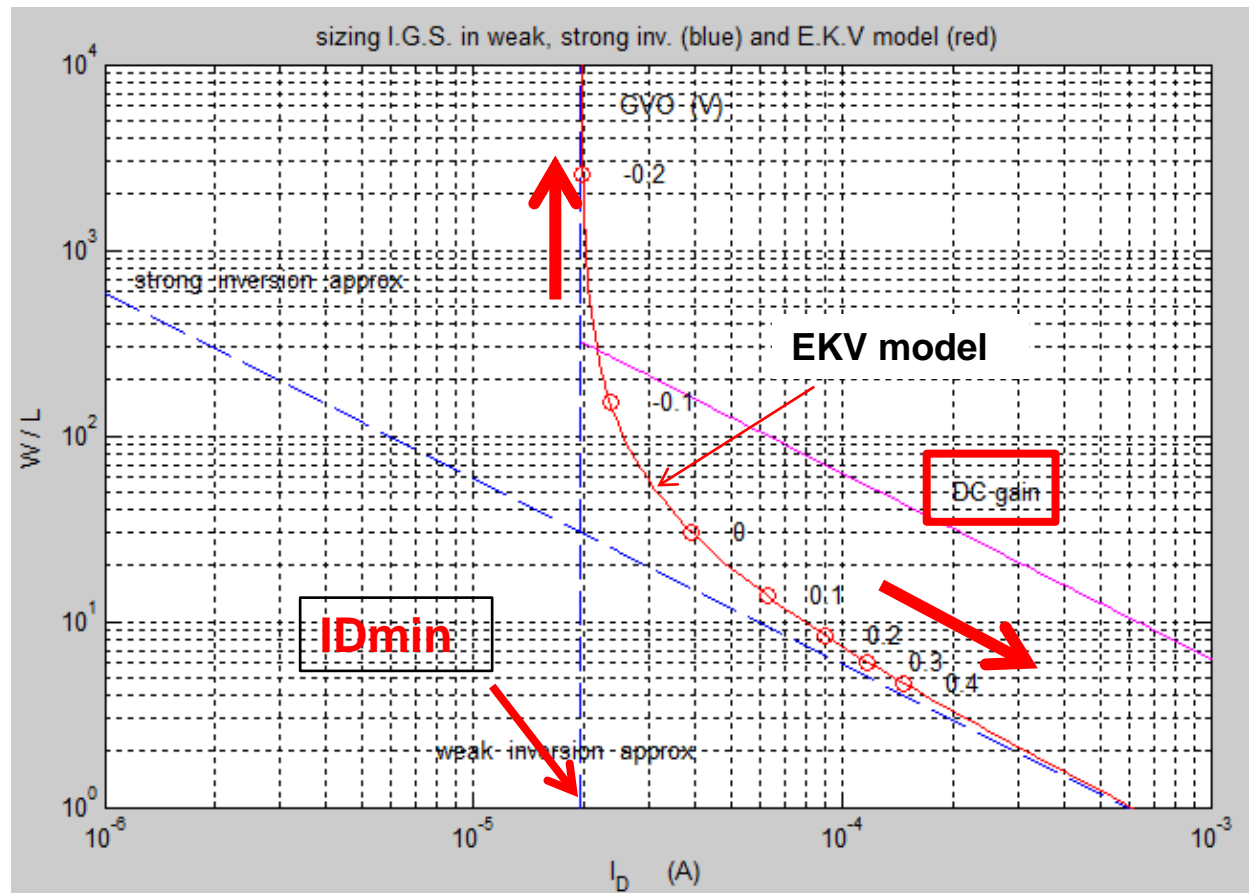
This figure displays
 (W/L) vs I_D
 achieving the
 desired GBW (i.e.,
 const f_T or g_m).

$$A_{DC} = -\frac{g_m}{I_D} V_A$$

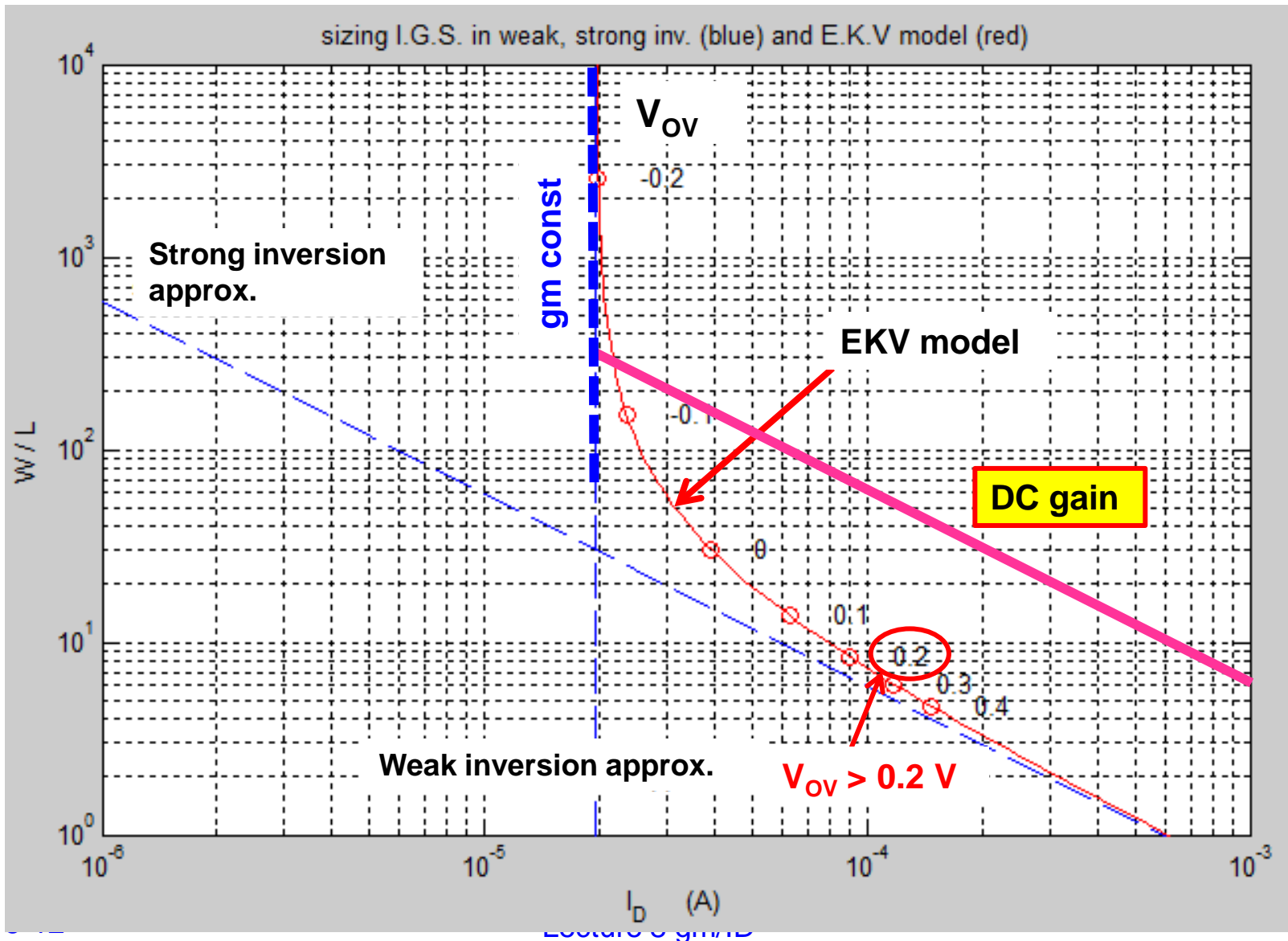
➔ for I_D large

$$\log A_{DC} = \log(-g_m V_A) - \log I_D$$

($g_m \sim \text{const}$)



W/L versus I_D (See comments next page)



Comment on the Figure

We see that the **DC gain varies like the reciprocal of I_D** ;

→ Smaller drain current, larger DC gain.

The largest DC gain is arrived at when I_D reaches the **minimum $I_{D,\min}$** .

The **DC gain is approximated by the equation** considering Early voltage:

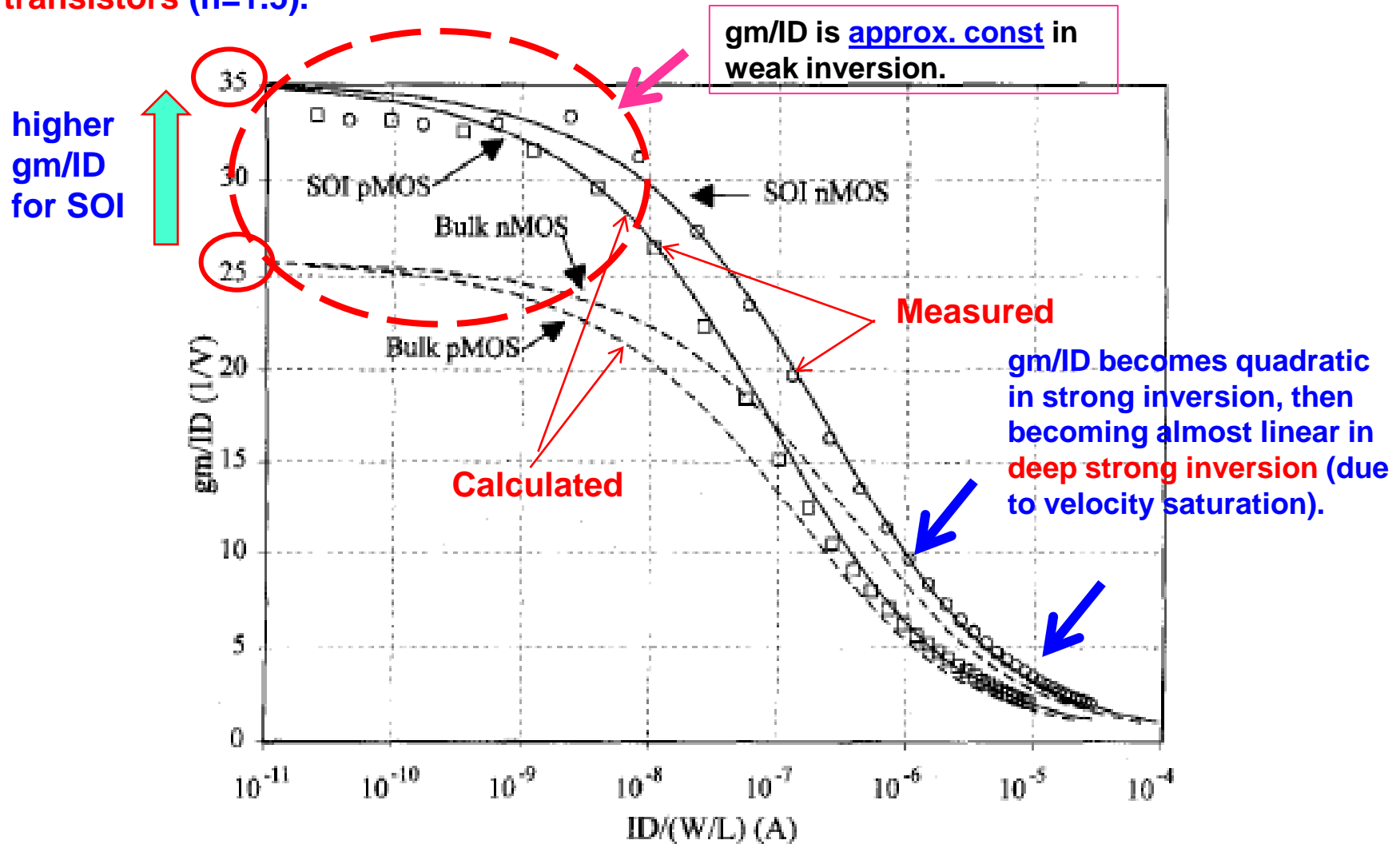
$$A_{DC} = -\frac{g_m}{I_D} V_A$$

When $I_D = I_{D,\min} = g_m n U_T$

→ the max DC gain:

$$A_{DC,\max} = -\frac{V_A}{n U_T}$$

The thin-film SOI transistors ($n=1.1$) has increased subthreshold slope (due to smaller n), giving a maximum value of gm/ID of about 35 while only 25 for bulk transistors ($n=1.5$).



Calculated and measured gm/ID vs $ID/(W/L)$ for bulk transistors and thin-film fully-depleted SOI transistors.

Observations

- gm/ID decreases when the normalized ID moves **toward the strong inversion region**.
- For the **same gm/ID**, $I_{D,p}$ is lower than $I_{D,n}$ due to the mobility difference.
 - Hence, **requiring larger W/L for pMOS** to achieve an equal ID.
- Hence, gm/ID is also an *indicator* of the transistor operation region.
- Both gm and ID are proportional to size;
- but **gm/ID is size independent**.
- Once any two values among **gm/ID**, **gm**, and **ID** are given, we can determine the aspect ratio **W/L**.

gm/ID vs normalized current

- The normalized current ($I_{\square} = ID/(W/L)$) is independent of the transistor size.
- The relationship between **gm/ID** and I_{\square} is a unique characteristic for one type of transistors.
 - However, this statement has to be revised when dealing with short channel transistors.
- The characteristic of (gm/ID) vs I_{\square} **can be explored extensively** during the design phase
- The actual gm/ID vs I_{\square} can be obtained by either analytical method (fitting) or measurement.

gm/ID Characterization

- Two characterization methods: semi-empirical or model-driven.
- **1) Semi-empirical:** it makes use of real measurements or data derived from advanced MOS models.
- **2) Model-based:** it applies simple models with reliable analytical expression.
 - The **basic EKV model** is a candidate **but not perfect**;
 - The EKV parameters are allowed to **vary with bias conditions and gate lengths**.

gm/ID Characterization

- In order to take into account of **process variations**,
- it is more appropriate to consider **averaged curves** which are representative of a large number of transistors

Accurate Reference Models

- **BSIM** is a widely used state-of-the-art model available in the public domain.
 - It is based on **threshold voltage formulations**;
 - But has weaknesses (model inaccuracy) in **moderate inversion**.
- **PSP** model from Penn State University and Philips (now NXP) is considered the more accurate industrial standard.
 - Based on the **surface potential model** (like the **Charge Sheet Model of EKV**).

gm/ID Sizing Procedure

gm/ID Sizing Procedure

We derived from the quadratic model:

$$\frac{g_m}{I_D} = \frac{2}{V_{GS} - V_{th}}$$

$$g_m \sim I_D \sim W$$

→ **(gm/ID) is independent of the gate width (W)**

- Determine g_m according to (GBW = f_T): $g_m = 2\pi f_T C$
- Determine I_D by the const gm/ID equation: $I_D = g_m / \left(\frac{g_m}{I_D}\right)^*$

The reference ratio $(gm/ID)^*$ is obtained from a similar device whose W^* and L^* are known.

- Determine the device size (W) by the proportionality btw I_D and W:

$$W = (W)^* \frac{I_D}{(I_D)^*}$$

The reference (gm/ID)*

The **reference (gm/ID)*** is defined by:

$$\left(\frac{g_m}{I_D}\right)^* = \frac{1}{I_D^*} \frac{dI_D^*}{dV_G} = \frac{d}{dV_G} \log(I_D^*)$$

Two typical methods:

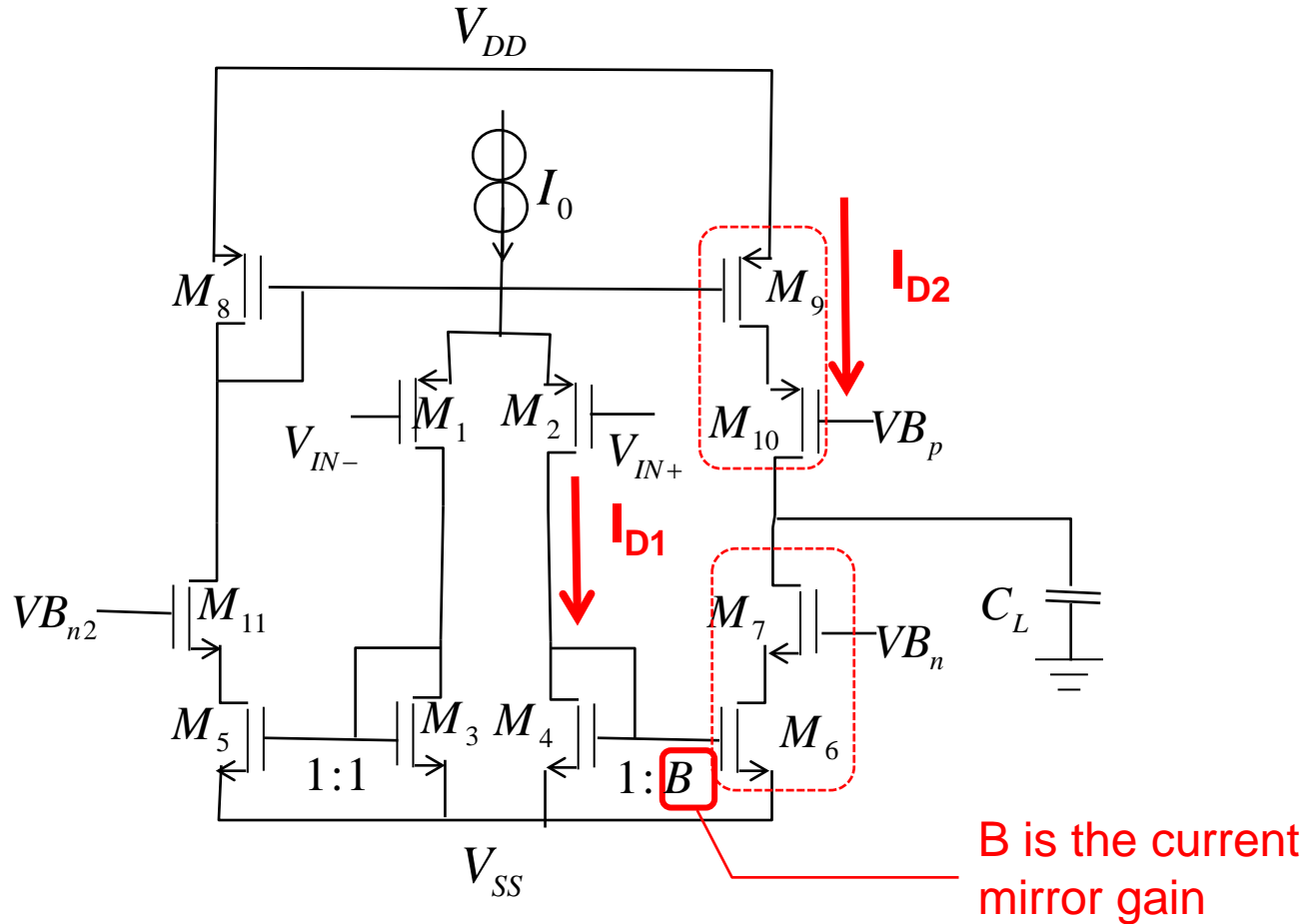
1. **Semi-empirical gm/ID sizing method**: Deriving the reference (gm/ID)* from **experimental $I_D(V_{GS})$ characteristics** (typically from advanced models such as BSIM or PSP).
2. **Model-based method**: Deriving (gm/ID)* from analytical large signal model **adequately accurate** by parameter fitting (e.g., EKV model).

Application to OTA Synthesis

Cascode OTA, CMOS-SOI technology

Application

Synthesize a cascode OTA by the gm/ID method:

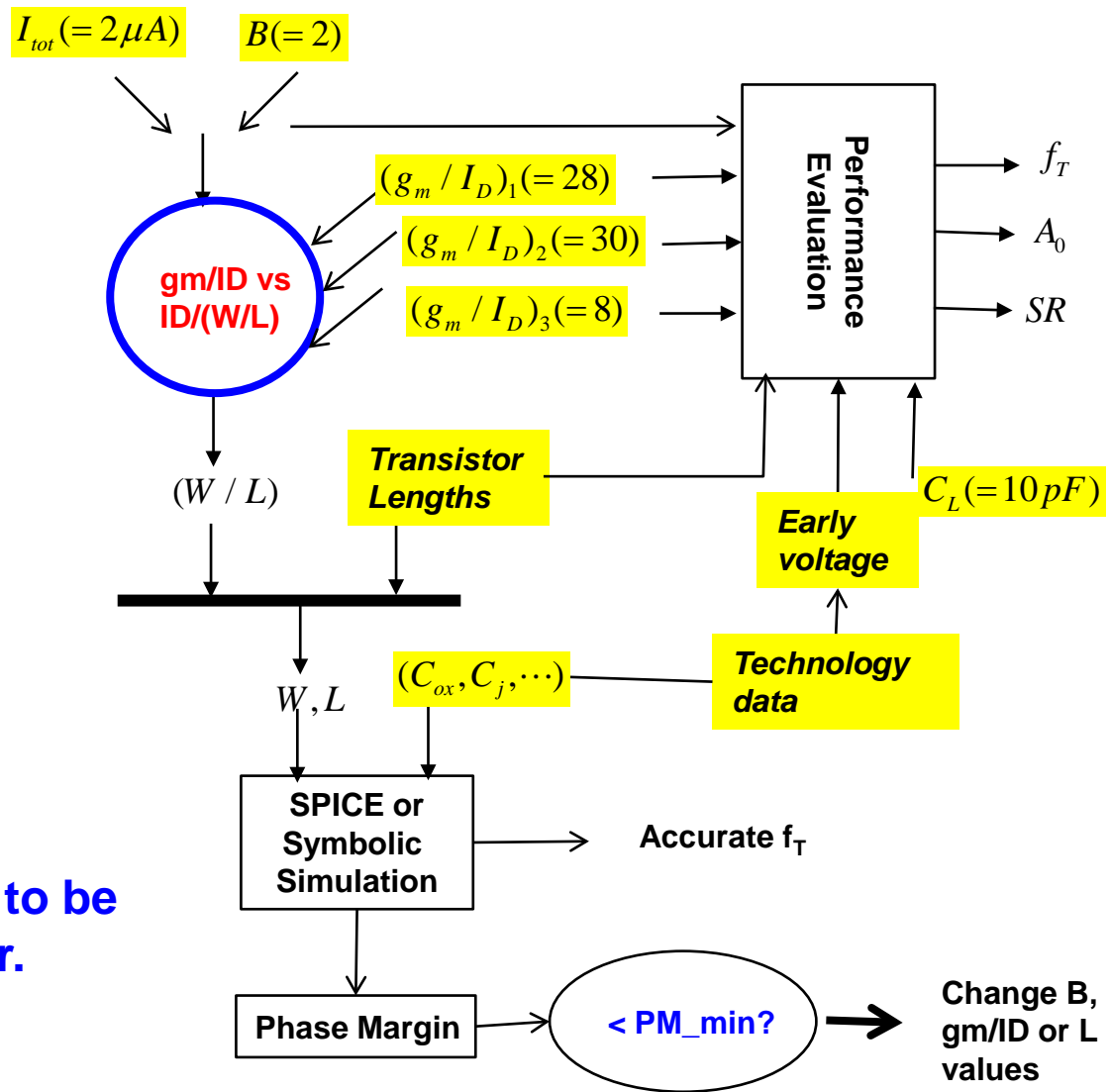


OTA schematic (to be implemented CMOS-SOI)

OTA Synthesis

- Assume total supply current is $I_{\text{tot}} = 2 \mu\text{A}$, load capacitor is $C_L = 10\text{pF}$, and supply voltage $V_{\text{DD}} = 3 \text{ V}$.
- Design to achieve the best performance of:
 - Open loop dc gain (A_{DC}),
 - Transition frequency (f_T),
 - Phase margin (PM), and
 - Slew rate (SR)
- It is straightforward to take into account of other performance aspects (like noise or common mode rejection) as long as they are directly related to the current and small-signal parameters.
- For **large-signal performance** such as **signal swing**, an “ I_D vs V_G ” or “ g_m/I_D vs V_G ” relationship is required.

gm/ID Design Flow



Highlighted are the data to be provided by the designer.

OTA Synthesis

The DC gain can be derived as:

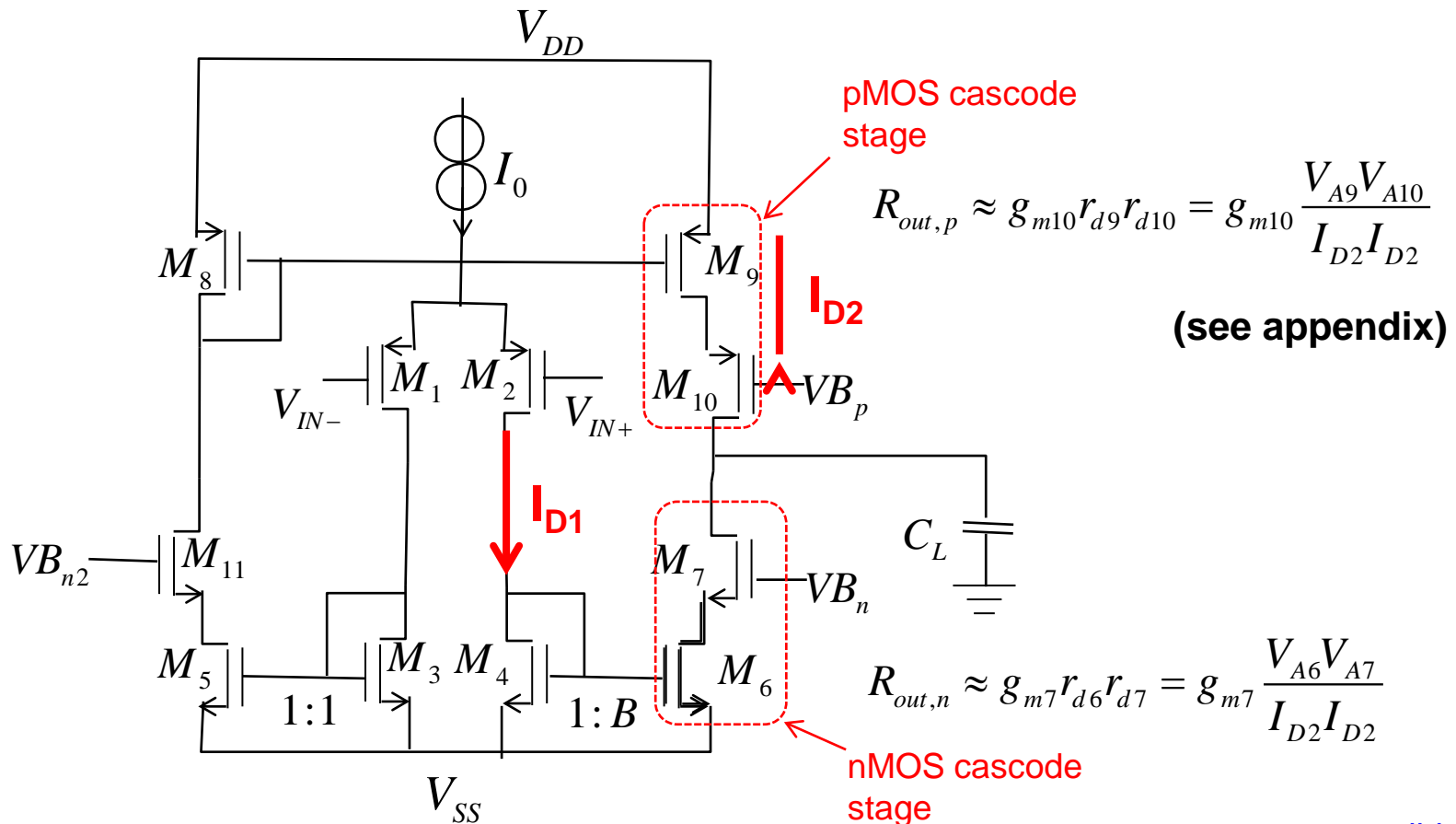
$$A_{DC} = \left(\frac{g_m}{I_D} \right)_1 \left(\frac{g_m}{I_D} \right)_2 \frac{1}{\frac{1}{V_{A6} \cdot V_{A7}} + \frac{1}{V_{A9} \cdot V_{A10}}} \quad (\text{to be derived next})$$

where $(g_m/I_D)_1$ is the ratio of the **input** transistors and $(g_m/I_D)_2$ is the ratio of the **cascode** transistors.
 $V_{A6,7,9,10}$ are Early voltages.

The **Early voltages** are considered proportional to the transistor length with a typical constant proportionality of **7V/um**. **(In the paper L is in the range of 3 to 12 um.)**

Derivation of DC Gain

$$A_{DC} = \left(\frac{g_m}{I_D} \right)_1 \left(\frac{g_m}{I_D} \right)_2 \frac{1}{\frac{1}{V_{A6} \cdot V_{A7}} + \frac{1}{V_{A9} \cdot V_{A10}}}$$



Derivation (cont'd)

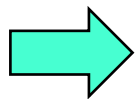
$$R_{out,p} \approx g_{m10} r_{d9} r_{d10} = g_{m10} \frac{V_{A9} V_{A10}}{I_{D2} I_{D2}}; \quad R_{out,n} \approx g_{m7} r_{d6} r_{d7} = g_{m7} \frac{V_{A6} V_{A7}}{I_{D2} I_{D2}}$$

$$R_{out} = R_{out,p} \parallel R_{out,n} \approx g_{m10} \frac{V_{A9} V_{A10}}{I_{D2} I_{D2}} \parallel g_{m7} \frac{V_{A6} V_{A7}}{I_{D2} I_{D2}}$$

$$= \frac{g_{m7,10}}{I_{D2} I_{D2}} \frac{1}{\frac{1}{V_{A9} V_{A10}} + \frac{1}{V_{A6} V_{A7}}}$$

$$g_{m7,10} = g_{m7} = g_{m10}$$

gain of 1st stage



$$A_{DC} = g_{m1} R_{out} = \frac{g_{m1} g_{m7,10}}{I_{D2} I_{D2}} \frac{1}{\frac{1}{V_{A9} V_{A10}} + \frac{1}{V_{A6} V_{A7}}}$$

$$= \left(\frac{g_m}{I_D} \right)_1 \left(\frac{g_m}{I_D} \right)_2 \frac{1}{\frac{1}{V_{A9} V_{A10}} + \frac{1}{V_{A6} V_{A7}}}$$

Assuming current mirror ratio from the input stage to the cascode stage is 1:1.

(cont'd)

$$SR = \frac{B \cdot I_{D1}}{C_L}$$

$$f_T = \frac{B \cdot g_{m1}}{2\pi \cdot C_L}$$

**1st order
approximation of f_T**

where **B** is the current mirror ratio, I_{D1} is the current of the first stage, and g_{m1} is the transconductance of the input transistor.

Set **B = 2** in the paper.

The max B value is limited by its influence on the OTA stability.

f_T is also proportional to gm/ID , given fixed ID .

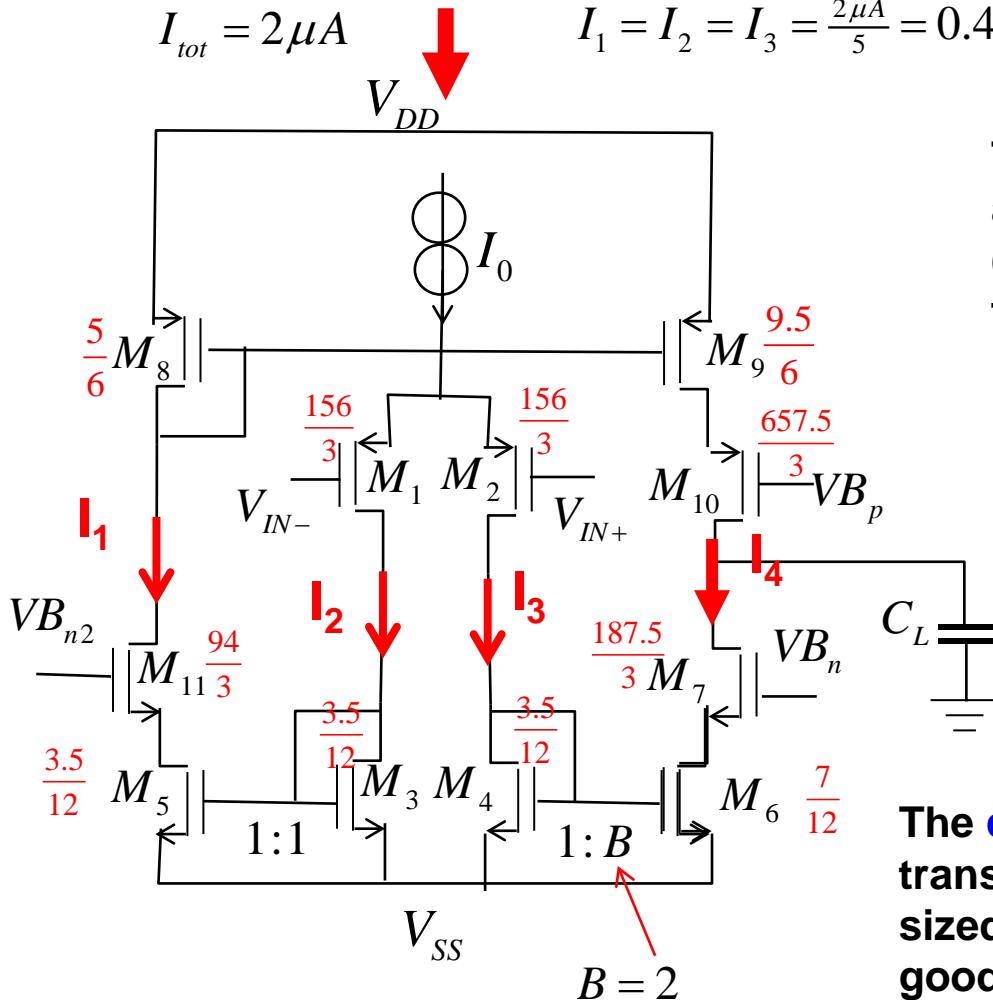
Sizing Procedure for the OTA

1. Determine the **drain current** of each transistor according to the total supply current and the current mirror ratio $B (= 2)$.
2. **Choose g_m/I_D** accordingly to their effect on the OTA **performance**,
3. Determine the normalized current according to **the experimental curve** of **(g_m/I_D) vs I_{\square}** .
4. Calculate W/L for each transistor.

Sizing the OTA

The total current is divided into four branch currents (**one branch doubled**):

$$I_{tot} = 2\mu A \quad I_1 = I_2 = I_3 = \frac{2\mu A}{5} = 0.4\mu A; \quad I_4 = B \cdot I_3 = \frac{4\mu A}{5} = 0.8\mu A$$



The g_m/I_D values are determined after **design space exploration** (optimizing tradeoff btw dc gain and f_T for a given PM).

$$\left(\frac{g_m}{I_D}\right)_{1,2} = 28; \quad \left(\frac{g_m}{I_D}\right)_{7,10} = 30;$$

(correspond to operation in the moderate inversion region close to W.I.)

The **current mirror** transistors (**M3,4,5**) are sized in **S.I.** to guarantee good matching and noise properties.

$$\left(\frac{g_m}{I_D}\right)_3 = 8;$$

Choice of g_m/I_D

- The dc gain is proportional to g_m/I_D → g_m/I_D higher better.
- f_T is proportional to g_m , hence g_m/I_D (if I_D is fixed).
→ also g_m/I_D higher better.
- However,
- The max g_m/I_D is limited by the weak inversion value
 - about 35 V^{-1} for thin-film fully-depleted SOI MOS (higher);
 - about 25 V^{-1} for bulk CMOS
- Also limited by the stability requirement
 - because increasing g_m increases the transistor sizes (W), (hence, the parasitic caps), reducing the phase margin.

Looking up I_{\square} from Curve

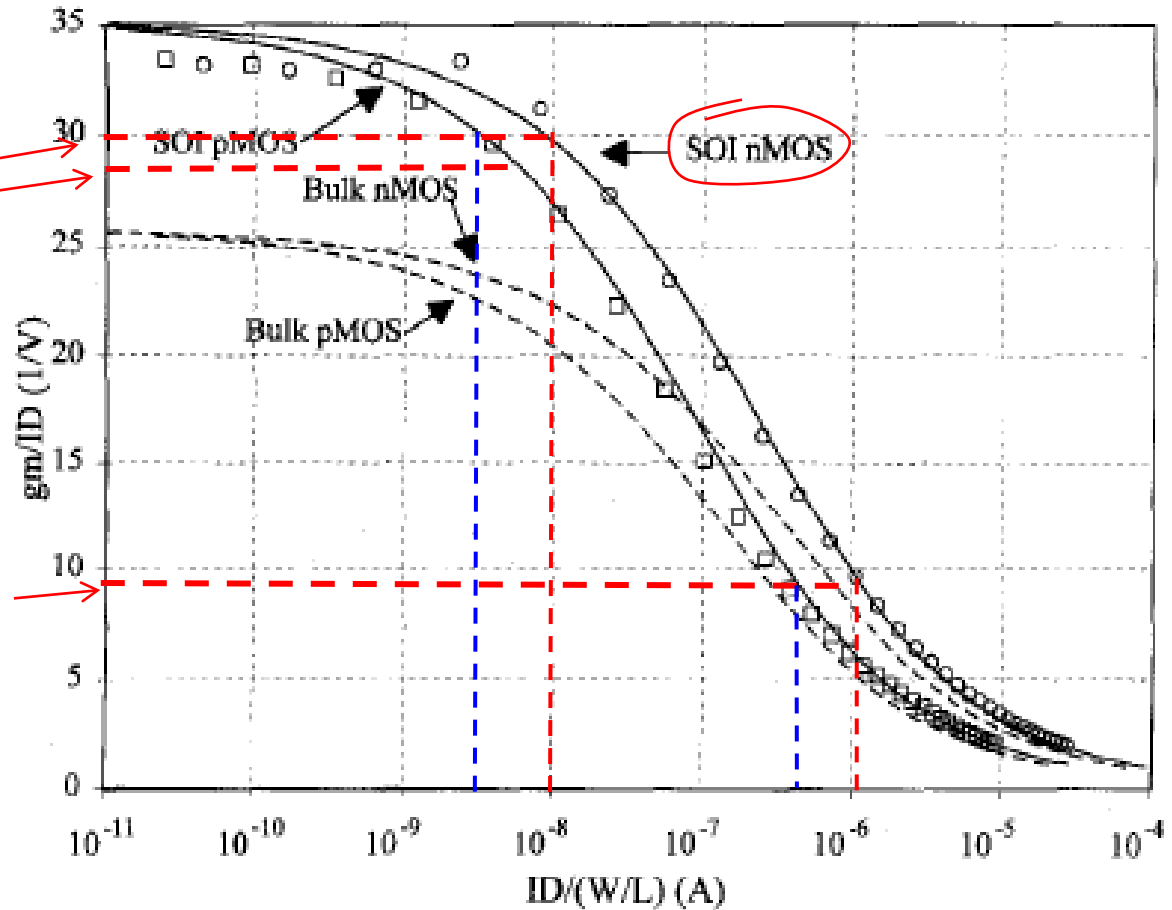
For nMOS, $I_{\square} \approx 10^{-8}$
 for $g_m/I_D = 30$.
 For pMOS, I_{\square} is
 smaller.

30

28

$\frac{g_m}{I_D}$

8

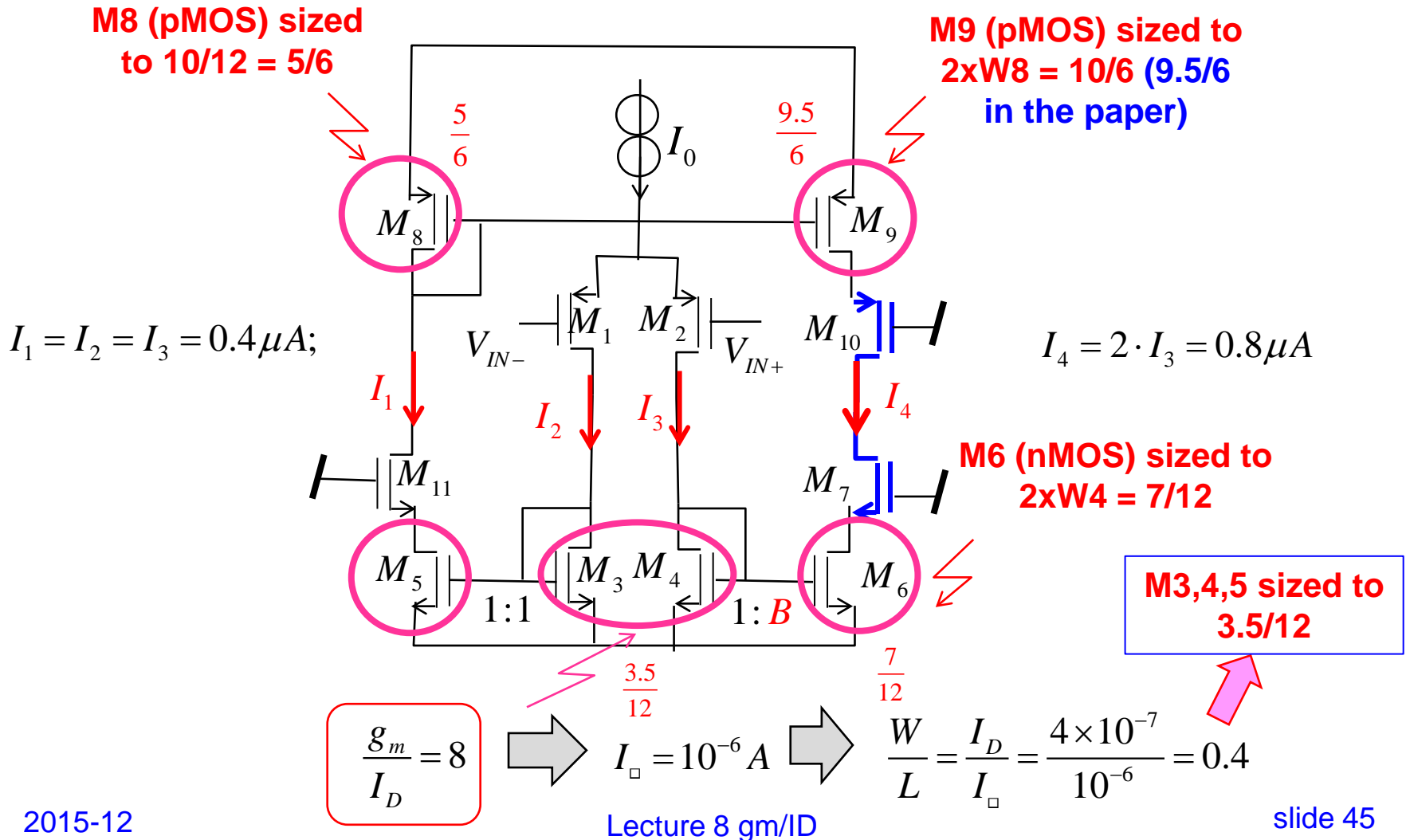


For nMOS, $I_{\square} \approx 10^{-6}$
 for $g_m/I_D = 8$.
 For pMOS, I_{\square} is
 smaller.

about 100x

Sizing Details - 1

1) Sizing the mirror transistors ...

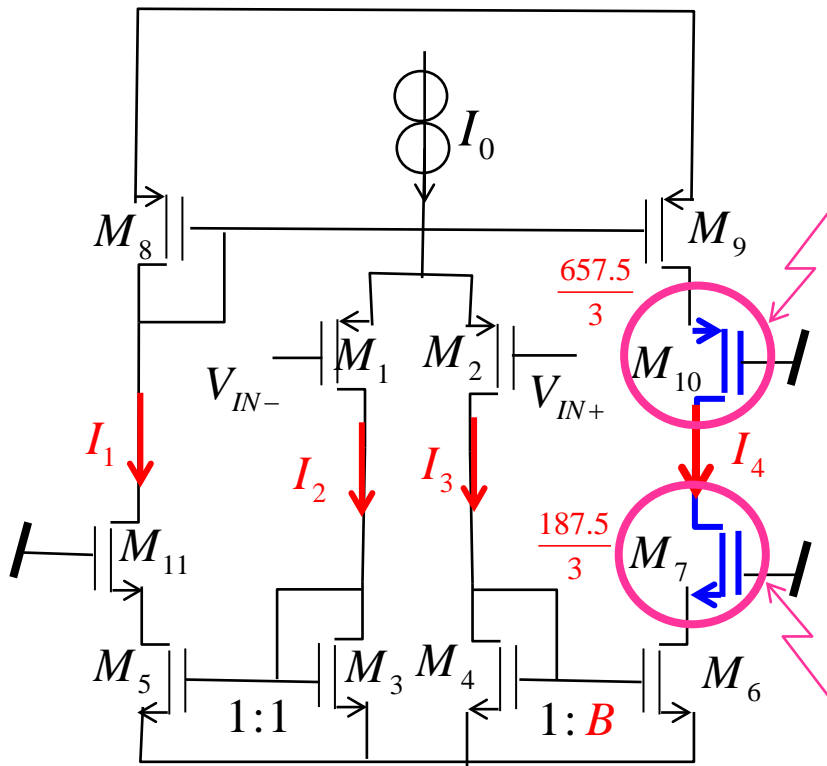


Sizing Details - 2

2) Sizing the cascode transistors (M7, M10) ...

$$I_1 = I_2 = I_3 = 0.4 \mu\text{A};$$

$$I_4 = 2 \cdot I_3 = 0.8 \mu\text{A}$$



**M10 (pMOS)
sized larger to
 $657.5/3 = 219$**

$$\frac{g_m}{I_D} = 30$$

**M7 sized to
 $187.5/3 = 62.5$**

$$I_{\square} = 10^{-8} \text{ A}$$

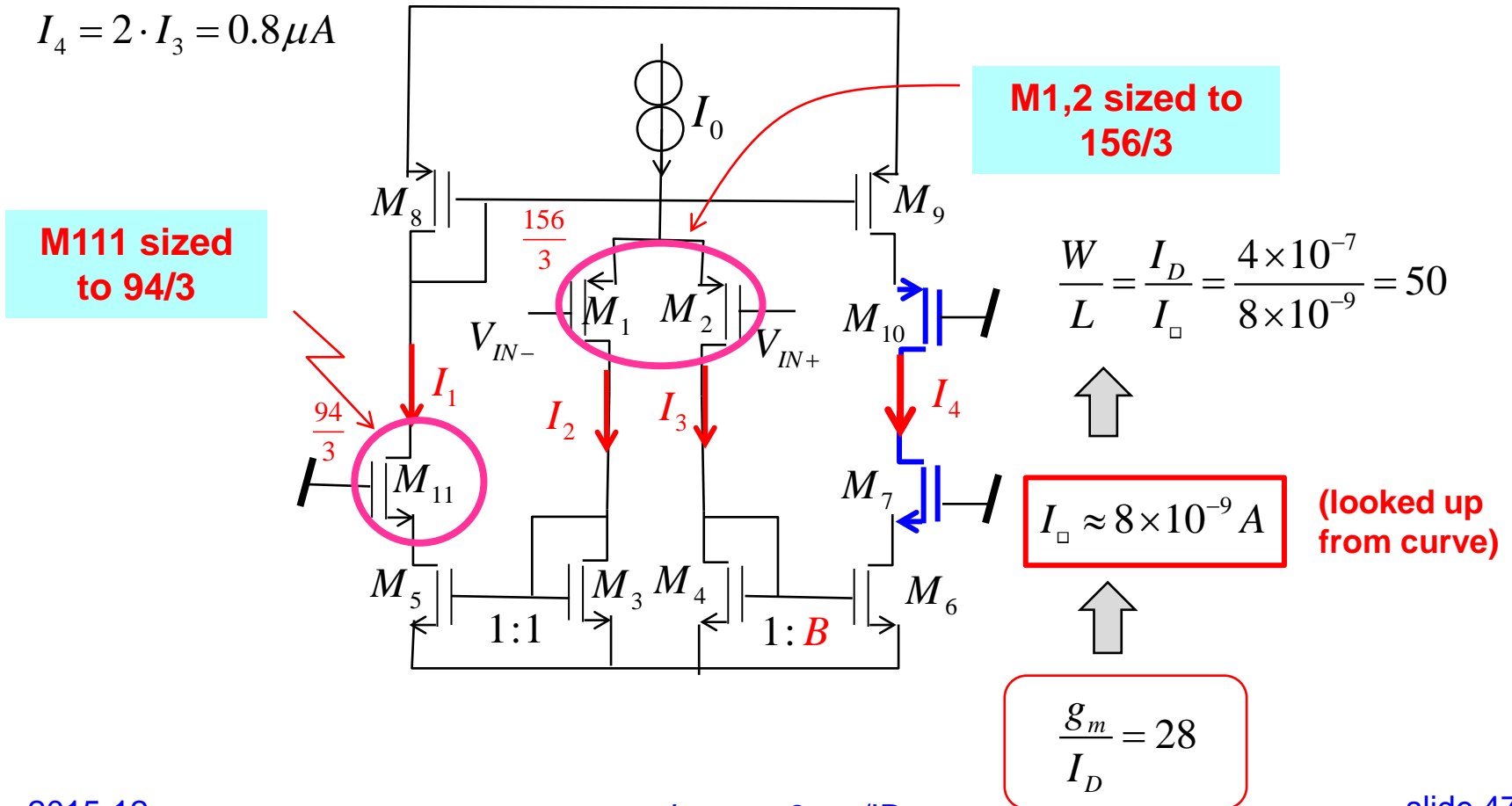
$$\frac{W}{L} = \frac{I_D}{I_{\square}} = \frac{8 \times 10^{-7}}{10^{-8}} = 80$$

Sizing Details - 3

3) Sizing the input transistors (M1,2) and M11 ...

$$I_1 = I_2 = I_3 = 0.4 \mu\text{A};$$

$$I_4 = 2 \cdot I_3 = 0.8 \mu\text{A}$$



Summary of Transistor Sizes

The transistor lengths are determined by a trade-off btw **area** and **stability**, and **dc gain** (dependence of Early voltage on L).
 $V_A \sim 7 \text{ (V/um)}$ L empirically for L 3~12um.

	W	L	Effective W/L
M1	156	3	57.6
M2	156	3	57.6
M3	3.5	12	0.26
M4	3.5	12	0.26
M5	3.5	12	0.26
M6	7	12	0.52
M7	187.5	3	69.3
M8	5	6	0.79
M9	9.5	6	1.58
M10	657.5	3	243.3
M11	94	3	34.6

Experimental Result

- The sized OTA was realized in the 3-um CMOS-SOI process.

TABLE II. Calculated, simulated and measured results of OTA

	Synthesis Prog.	HSPICE	Measurements	Notes
$(gm/ID)_1$ (1/V)	28	29.4	28.3	
$(gm/ID)_2$ (1/V)	30	31.6	30.5	
A0 (dB)	103.9	105.5	103	
f_T (kHz)	324	336		@CL=10pF
PM (deg.)	72.5	72		@CL=10pF
f_T (kHz)	261*	270*	271*	@CL=12.3pF
PM (deg.)	63.8*	63*	60*	@CL=12.3pF
SR (V/us)	0.11	0.09	0.1	@CL=12.3pF
Output swing (Vpp)	2.02	2.2	1.93	@Vdd=3V

HSPICE uses level 34 model with a set of parameters optimized to fit the SOI MOSFET characteristics.

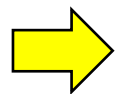
DC Gain Estimation

The open-loop dc gain can be estimated as follows:

$$A_{DC} = \left(\frac{g_m}{I_D}\right)_1 \left(\frac{g_m}{I_D}\right)_2 \frac{1}{\frac{1}{V_{A6} \cdot V_{A7}} + \frac{1}{V_{A9} \cdot V_{A10}}}$$

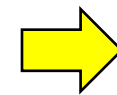
$$\left(\frac{g_m}{I_D}\right)_1 = 28V^{-1}; \quad \left(\frac{g_m}{I_D}\right)_2 = 30V^{-1}; \quad V_A \approx (7V/\mu m) \cdot L$$

$$L6 = 12; L7 = 3; L9 = 6; \\ L10 = 3 \mu m$$



$$A_{DC} = 49 \times (28)(30) \frac{1}{\frac{1}{12 \times 3} + \frac{1}{6 \times 3}}$$

$$= 18 \times 49 \times (28)(30) \times \frac{2}{3} = 493,920$$



$$20 \log_{10}(A_{DC}) = 113.8 \text{ dB}$$

~ HSPICE 105.5 dB

Comparison to Other Synthesis Methods

TABLE III. Comparison of results of gm/ID based synthesis with conventional **strong inversion (SI)** and **weak inversion (WI)** synthesis

	gm/ID method	SI synthesis.	SI real	WI synthesis.	WI real
$(gm/ID)_1$ (1/V)	28	28	18.7	35	30
$(gm/ID)_2$ (1/V)	30	30	19.7	35	30.5
A_0 (dB)	103.9	103.9	96.7	107.1	104.6
f_T (kHz)	324	351	236	395	344
PM (deg.)	72	84	86	64	68
W/L input pair	57.6	7.7	7.7	120.9	120.9
W/L cascode n	69.3	6.47	6.47	93.5	93.5
W/L cascode p	243.3	17.3	17.3	241.7	241.7
$\Sigma W \cdot L$ (μm^2)	4900	1359	1359	6185	6185

Die area



By **strong inversion (SI)** synthesis, it overestimates f_T by about 50% and the gain by about 7dB.
By **weak inversion (WI)** synthesis, it overestimates f_T by 15% and the die area by 25%.

Comments on the comparison

- The **strong inversion (SI)** synthesis **extends quadratic expression to moderate inversion**, this will *underestimate* transistor size (W/L), hence amplifier area.
- The **weak inversion (WI)** synthesis considers the **exponential approximation** for I_D versus V_G , it predicts the g_m/I_D equals to 35, independent of the current. The $I_D/(W/L)$ cannot be determined, it is chosen to guarantee weak inversion operation.
- Using the transistor sizes provided by the SI and WI synthesis, then referring to the real g_m/I_D versus $I_D/(W/L)$ data, the resulting estimations are listed as “**SI real**” and “**WI real**”.

Summary

- gm/ID creates a connection btw the **transconductance** gm (a small-signal quantity) to the **drain current** ID (a large-signal quantity).
- The gm/ID methodology provides a unified sizing procedure for MOS devices from the *strong to the weak inversion region*.
- The gm/ID sizing methodology applies as long as the **widths are large enough** so that the lateral effects can be ignored,
 - A condition that holds with most CMOS analog circuits.
- The OTA sized with this methodology results in lower current consumption with increased gain given a bandwidth.

References on gm/ID

- **F. Silveira, D. Flandre, P. Jespers, “A gm/ID based methodology for the design of CMOS analog circuits and its application to the synthesis of a silicon-on-insulator micropower OTA,” IEEE J. Solid-State Circuits, vol. 31, no. 9, Sept 1996, pp. 1314–1319.**
- **P. G. A. Jespers, The gm/ID Methodology, A Sizing Tool for Low-Voltage Analog CMOS Circuits, The Semi-empirical and Compact Model Approaches, Springer, Heidelberg, 2010.**
- **D. M. Binkley et al, “A CAD methodology for optimizing transistor current and sizing in analog CMOS design,” IEEE Trans. CAD, vol. 22, no. 2, Feb. 2003, pp. 225-237.**
- **D. M. Binkley, Tradeoffs and Optimization in Analog CMOS Design, Wiley, Chichester, England, 2007.**
- **A. Girardi, F. P. Cortes, and S. Bampi, “A tool for automatic design of analog circuits based on gm/ID methodology,” IEEE ISCAS, 2006.**

References on EKV

- E. A. Vittoz and J. Felrath, “CMOS analog integrated circuits based on weak inversion operation,” *IEEE J. Solid-State Circuits*, vol. 12, no. 3, pp. 224-231, June 1977.
- E. A. Vittoz, “Low-power low-voltage limitations and prospects in analog design,” in *Proc. Workshop Advances in Analog Circuit Design*, Eindhoven, Mar. 1994.
- A good webpage on EKV by Enz:
<http://www.ieee.org/portal/pages/sscs/08Summer/Enz.html>

Project Assignment

- Use **0.18 um (bulk or SOI)** to design the OTA presented in this lecture.
- Use **HSPICE level 34 model** to fit the gm/ID vs I_{DQ} curve.
- **Size the transistors to achieve the maximum possible performance.**
- **Discuss the performance trade-offs:**
 - Gain, transition frequency (f_T), PM, slew rate (SR)
 - Power and area
 - Noise