New approaches to interconnect macromodeling with explicit delay extraction *

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Abstract— **IC design automation relies on macromodels for interconnect analysis. For simulation speed, low-order macromodels are in general preferred. Besides rational macromodels, explicit delay components $e^{-\tau s}$ are of special interest in interconnect timing. This paper investigates new approaches to modeling interconnects in the form of multiplying a rational function by a delay component. Fitting-based techniques are used both in the time-domain and in the frequency-domain for the purpose of hybrid modeling using sampled data. A technique for passivity test is also introduced. Examples are given to demonstrate the effectiveness of the proposed methodology.**

I. INTRODUCTION

Since the publication of the widely cited AWE paper [1], vast amount of research effort in the CAD community has been dedicated to rational modeling of interconnect systems in the frequency domain. Specifically, one would like to derive a rational form of transfer function

$$H(s) = \frac{\beta(s)}{\alpha(s)} = \frac{b_0 + b_1 s + \cdots + b_m s^m}{a_0 + a_1 s + \cdots + a_n s^n}, \qquad (1)$$

which approximates the input-output behavior of any given linear interconnect model with adequate accuracy in the frequency domain. Issues related to this goal include: 1) How to *compute* the coefficients in the rational function efficiently and robustly? 2) How to guarantee the *stability* and *passivity* of the derived model?

One may derive a low-order rational model either from a high-order *state-space model*, or from a set of *sampled data* describing the input-output characteristic of an interconnect system. Direct model order reduction from a state-space model was considered solved after the publication of the paper [2], while the computation issue related to the sampled data modeling still receives quite some attention. Problems regarding stability and passivity guarantee are not considered well-solved due to the the extra computations involved.

To further speed up simulation, one would normally choose the model order n ($n \geq m$) in (1) as low as possible, as long as the model does not lose the required accuracy in application. Recently, it has been widely recognized that propagation time over interconnect in IC design is becoming more significant as the silicon feature size further shrinks. This has motivated many authors to consider extracting the delay element explicitly before deriving a rational model. This treatment would in general bring two advantages: one for delay characterization such as in critical paths or clock networks, and the other for simulation speed up. An early work on frequency dependent transmission line models with explicit delay elements was presented in [3]. Research on lossy transmission line modeling using hybrid phase-pole macromodel was proposed in [4]. The authors of [6] proposed analytical formula based method for estimating line delay; they derived delay formula from the moments of a second order circuit, then extended this formula to arbitrary order circuits.

An effective methodology for delay extraction remains a debatable issue. This paper proposes a delay identification method based on the extraction of non-minimum phase zeros and a two-round fitting procedure using sampled data for hybrid modeling.

II. CONCEPTUAL ALGORITHM

We restrict our discussion to single-input and single-output (SISO) linear system. Any such system can be represented in state-space form as

$$\begin{cases} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= c^{\mathrm{T}}x(t) + du(t) \end{cases} \qquad (2)$$

where $x(t) \in \mathbb{R}^N$ is a column vector denoting the state, $u(t)$ and $y(t)$ are scalar input and output, respectively. Both b and c are column vectors of the same dimension as $x(t)$. d is a scalar representing the feed through gain. It is generally assumed that the system dimension N is so large that directly using this model for simulation would not be efficient. Hence, it has been a common practice to develop another approximate low-order linear model as a substitute for simulation.

Because of its convenience and viability, most *rational* reduced order modeling techniques derive a *rational* model in the form of (1) from the original state-space model, which is more or less a mature subject. A less mature subject is to derive from a given linear system (2) (or a set of measured data) a *hybrid* model of the following form

$$H(s) = e^{-\tau s}\frac{\beta(s)}{\alpha(s)} = e^{-\tau s}\frac{b_0 + b_1 s + \cdots + b_m s^m}{a_0 + a_1 s + \cdots + a_n s^n}, \qquad (3)$$

where $e^{-\tau s}$ represents a delay term. This task becomes more challenging if the original system is multi-input and multi-output (MIMO). The difficulty arises from the fact that the unknown delay τ appears nonlinearly with the unknown polynomial coefficients b_i's and a_i's, hence cannot be identified all together *simultaneously*. This issue becomes more intricate for the MIMO models. However, the advantage of having a model in the hybrid form (3) is that the rational model order n is likely to be much lower and the explicit delay τ can directly be used for timing.

The key idea of this paper comes from a basic observation on the delay term $e^{-\tau s}$. For $s = j\omega$, we have $|e^{-\tau j\omega}| = 1$ for all ω, i.e., the *all-pass* property. If approximating $e^{-\tau s}$ by an all-pass *rational* function, we may choose a stable rational function in the following form

$$e^{-\tau s} \approx \frac{P_K(s)}{Q_K(s)} = \frac{\prod_{i=1}^{K}(s + q_i^*)}{\prod_{i=1}^{K}(s - q_i)} \triangleq \varepsilon(s), \qquad (4)$$

where * indicates complex conjugate and all $\mathrm{Re}(q_i) < 0$. Substituting the rational expression for $e^{-\tau s}$ into the hybrid form (3) results in another purely rational function. Hence, the problem of deriving a hybrid macromodel (3) can be solved in the following steps:

Hybrid Macromodeling Algorithm

Step 1: Use any rational modeling technique to derive an approximate rational model of the original system.

Step 2: If the identified rational model has zeros in the *right-half* complex plane, then collect all such non-minimum phase zeros and form a rational expression in the form of (4) as an approximation of the delay term $e^{-\tau s}$.

Step 3: Remove the delay term $e^{-\tau s}$ from the original model (or data) and do another round of rational modeling to get a lower-order rational model.

Step 4: Combine the two parts identified in the preceding two steps to obtain a hybrid model.

Step 5: Check stability and passivity. Repeat the procedure if needed.

III. REVIEW OF FITTING ALGORITHMS

There are two major approaches to rational modeling in the literature; model-based and data-based. The methodology developed in this paper is *data-based*; i.e., our hybrid models are derived from the sampled data either in the time-domain or in the frequency-domain.

Fitting techniques have been widely used in data-based rational modeling. In this paper we choose a special type of iterative fitting algorithms, the *Vector Fitting* (VF) algorithm [7], a reformulation of the Sanathanan-Koerner algorithm ([8], [9]), and its time-domain variant *Time-Domain Vector Fitting* (TDVF) [10]. We shall use examples to demonstrate that both algorithms are applicable for hybrid macromodeling with explicit delay extraction.

The VF algorithm assumes that a frequency response function $H(s)$ can be approximated by a rational function in expanded partial fractional form

$$H(s) \approx \sum_{i=1}^{n} \frac{c_i}{s - a_i} + d, \tag{5}$$

where $\{a_i\}$ and $\{c_i\}$ are unknown poles and residues, respectively, and term d is an unknown constant. (We have dropped the derivative term hs in (5) for simplicity.)

The VF or SK algorithm is to iteratively solve a sequence of linear least-squares problems, where in each step the poles are identified first and the residues are identified followed by. A weighting factor in rational form

$$\sigma(s) = \sum_{n=1}^{n} \frac{\overline{c}_i}{s - \overline{a}_i} + 1 = \frac{\prod_{i=1}^{N} (s - \overline{z}_i)}{\prod_{i=1}^{N} (s - \overline{a}_i)}, \tag{6}$$

is multiplied to the given frequency response so that the following equality holds for the given frequency points in the least square sense

$$\sigma(s)H(s) = \sum_{i=1}^{n} \frac{c_i}{s - \overline{a}_i} + d. \tag{7}$$

This identity implicitly assumes that the *zeros* of $\sigma(s)$ are identical to the *poles* of the rational representation of $H(s)$. Since the poles are known (or arbitrarily chosen in the first step) in (7), the unknown residues can be identified easily by linear least squares.

The formulation of Time-Domain Vector Fitting (TDVF) [10] assumes that a pair of input excitation $x(t)$ and output response $y(t)$ is known satisfying the input-output relationship $Y(s) = H(s)X(s)$. Then, according to (7), the following equation holds

$$\sigma(s)Y(s) = \left\{ \sum_{i=1}^{n} \frac{c_i}{s - \overline{a}_i} + d \right\} X(s), \tag{8}$$

which takes the following form in the time-domain

$$y(t) + \sum_{i=1}^{n} \overline{c}_i y_i(t) = dx(t) + \sum_{i=1}^{n} c_i x_i(t), \tag{9}$$

where the sequences $y_i(t)$ are defined by the convolutions

$$y_i(t) = \int_{0}^{t} e^{\overline{a}_i(t-\tau)} y(\tau) d\tau \tag{10}$$

and similarly for $x_i(t)$. The unknowns in (9) are solved again by a least squares procedure using the data sampled from the signals $x(t)$ and $y(t)$ at the selected time points t_k. The zeros solved for $\sigma(s)$ in the current step are used as the poles in both (7) and (9) in the next step, then another round of residual identification is performed until adequate convergence is reached.

IV. DATA-BASED HYBRID MODELING

Using the fitting algorithms for rational modeling as outlined in the *Hybrid Macromodeling Algorithm*, we present in this section the detailed computations required.

A. Delay identification

VF or TDVF identifies a rational representation of $H(s)$ in the form of (5). This frequency response function can easily be written in state-space form as given in (2) with $A = \text{diag}\{a_1, \cdots, a_n\}$, $b^{\mathrm{T}} = (1, \cdots, 1)$ (an all-one vector), and $c^{\mathrm{T}} = (c_1, \cdots, c_n)$. The zeros of this system can be obtained from the poles (eigenvalues) of the following reversed system

$$\dot{\xi} = \hat{A}\xi + bd^{-1}y \tag{11}$$

where $\hat{A} = A - bd^{-1}c^{\mathrm{T}}$.

With fine tuning, both VF and TDVF algorithms are able to derive stable rational functions with certain passivity enforcement [11], [12].

In experiments we found that for transmission lines with significant delay, the rational functions derived from fitting exhibited *non-minimum phase zeros* in the right-half plane. By identifying all non-minimum phase zeros, an all-pass rational function $\varepsilon(s)$ is constructed as stated in section II. Then the delay τ is calculated from the phase information of $\varepsilon(s)$ at the frequency sample points ω_k by a least squares optimization

$$\phi_k = -\omega_k \tau, \tag{12}$$

where $\phi_k = \text{phase}\{\varepsilon(j\omega_k)\}$.

Once the delay τ is obtained, the hybrid transfer function approximation becomes $H(s) \approx \overline{H}(s)e^{-\tau s}$, where $\overline{H}(s)$ is a rational function. It is in the frequency-domain equivalent to

$$X(s)\overline{H}(s) = e^{\tau s}Y(s), \tag{13}$$

or in the time-domain equivalent to

$$x(t) * \overline{h}(t) = y(t + \tau). \tag{14}$$

Then the second round of VF or TDVF fitting re-identifies the rational part $\overline{H}(s)$. It is expected that the order of $\overline{H}(s)$ be lower than that of $H(s)$ with comparable fitting rms-errors.

B. Passivity enforcement

The work in [11], [12] developed algorithms to enforce passivity of rational fitting models using either Quadratic Programming(QP) in residue perturbation or Hamiltonian matrix perturbation. These treatments only have limited passivity enhancement but with demanding computations; in general the QP method does not guarantee the passivity over the full frequency range.

Recently, another passivity test was proposed in the control literature [13], [14]. For the state-space equation stated in (2), the transfer function of this system is

$$H(s) = d + c^{T}(sI - A)^{-1}b. \tag{15}$$

According to this test, $H(s)$ is strictly positive real(SPR), i.e., strictly passive, if and only if: 1) $d > 0$; 2) A is stable; and 3) the matrix $(A - (1/d)bc^{T})A$ has no eigenvalues on the closed negative real axis $(-\infty, 0]$. Conditions 1) and 2) are relatively easy to enforce during a fitting process but for condition 3). Nevertheless, condition 3) can be used to test whether the rational models produced by fitting (even with passivity enforcement) are truly passive.

V. Experimental Results

In this section, two circuits are used to demonstrate the applicability of the proposed hybrid macromodeling algorithm in interconnect analysis. We mainly checked the following questions in our experiment: 1) Whether is the delay extracted comparable to the delay measured from the waveform [6]? 2) After the delay is extracted, whether can the order of the rational component further be reduced? and 3) Whether do the hybrid models finally obtained remain stable and passive?

Circuit 1 consists of three coupled parallel wires, each wire is modeled by nine RLC segments with capacitive coupling. Circuit 2 is an H-shape clock-tree with four terminal branches, each wire segment is modeled by ten RLC stages. The MNA formulations of the two circuits result in full-order models of order 89 and order 213, respectively.

TABLE I
RESULTS OF 1ST ROUND FITTING WITH DELAY EXTRACTION.

Circuit	VF			TDVF		
	order (iter)	rms-error	τ	order (iter)	rms-error	τ
1	18(4)	3.83E-2	7.79E-11	16(4)	3.25E-2	8.76E-11
2	16(5)	2.87E-6	2.30E-10	18(4)	4.15E-2	2.33E-10

The orders listed in Table I are the rational model orders used in the first round of fitting for the purpose of delay extraction. The iteration times needed by using VF and TDVF are also listed in the table. The rms errors measure the fitting accuracy at the end of first round fitting. The delays (τ) listed are extracted from the non-minimum phase zeros. It is worth noting that inductive circuits usually exhibit appreciable initial delays before the step responses start to rise. For the circuits under experiment, we used HSPICE to measure the initial delay at the output during the transient from $0\,V$ to $1/100\,V$; the delays observed for circuits 1 and 2 were respectively $7.32E-11\,s$ and $2.58E-10\,s$ (very close to what we estimated.) As a comparison, the delays estimated using the method in [6] for circuit 1 and 2 were respectively $9.92E-11\,s$ and $3.56E-10\,s$. Figs. 1, 2, 3, and 4 show the fitting results produced by VF and TDVF in the frequency-domain and time-domain, respectively.
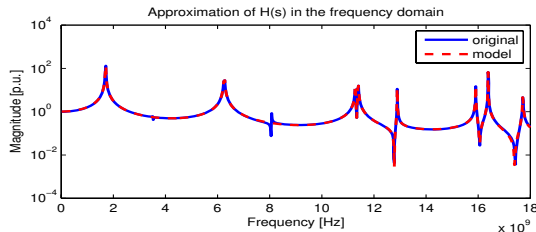

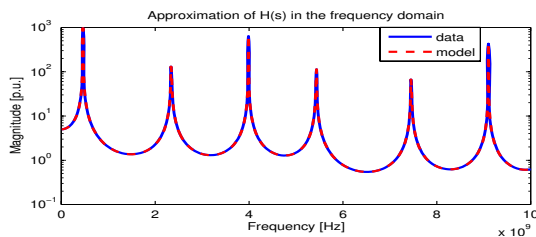Fig. 1.   First round fitting result of Circuit 1 using VF.

Fig. 2.   First round fitting result of Circuit 2 using VF.

In these two examples, we use 800 linearly distributed sampled points to do the VF and TDVF. In experiment, we found that both VF and TDVF algorithm are robust enough to converge in few times
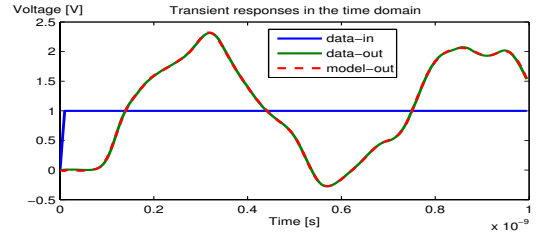

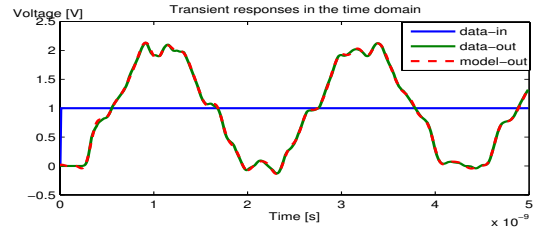Fig. 3.   First round fitting result of Circuit 1 using TDVF.

Fig. 4.   First round fitting result of Circuit 2 using TDVF.

of iteration even using fewer sample points. And in comparison with the data listed in Table III, both VF and TDVF could use lower order macromodels than famous non-hybrid model order reduction technique PVL[2] to model the circuit with comparatively equal or even lower rms-error. Another advantage of the fitting methods, say VF and TDVF, is that it does not need to know the structure of the original circuit and do the time consuming Krylov subspace projecting process, it only needs a few sampling points of the behavior of the original circuit.

Shown in Table II are the second round fitting results with new choices of the rational model order after the delay terms are removed. We found in experiment that, by keeping the fitting accuracy comparable to the first round fitting, the TDVF could result in a lower order rational component in the second round of fitting than the VF method; the exact reasons are not clear to the authors at the moment. This phenomenon is also observed by the authors in [4] using the time domain hybrid phase-pole macromodeling technique. However, the phase delay τ is also useful not only in the time domain but also in the frequency domain[5]. Shown in Figs. 5, 6, 7, and 8 are the rational fitting results of the second round using VF and TDVF.
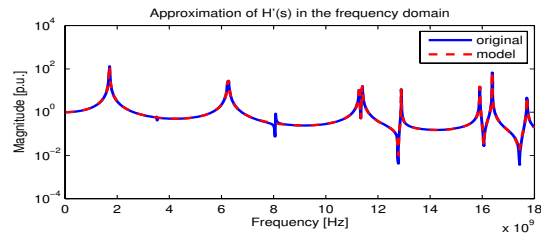

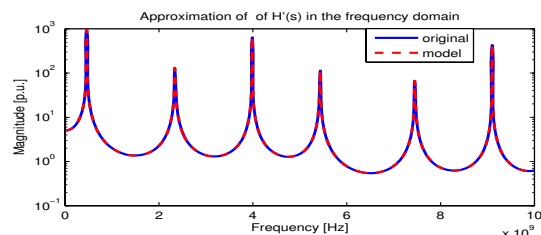Fig. 5.   Second round fitting result of Circuit 1 using VF.

Fig. 6.   Second round fitting result of Circuit 2 using VF.

TABLE II
RESULTS OF 2ND ROUND FITTING.

Circuit	VF		TDVF	
	order(iter)	rms-error	order(iter)	rms-error
1	18(4)	3.92E-2	6(4)	6.12E-2
2	14(3)	6E-4	8(4)	8.30E-2

TABLE III
RESULTS OF MODEL ORDER REDUCTION USING PVL.

Circuit	PVL	
	order	rms-error
1	41	3.04E-2
2	44	2.00E-2

TABLE IV
PASSIVITY TEST OF THE HYBRID MODELS

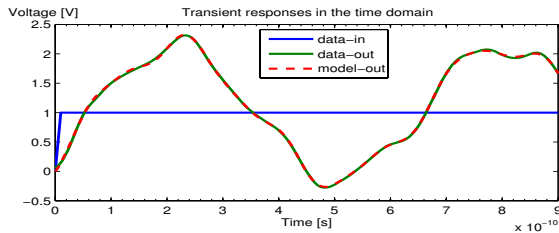| Circuit	VF		TDVF	
	d	passivity	d	passivity
1	1.21E-2	Yes	4.61E-2	Yes
2	5.00E-4	Yes	2.32E-1	Yes
τ-extracted 1	8.03E-3	Yes	-5.81E-2	No
τ-extracted 2	2.26E-2	Yes	-3.75E-2	No

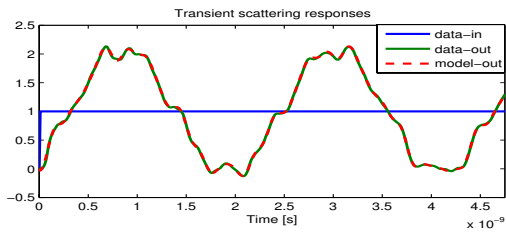Fig. 7.   Second round fitting result of Circuit 1 using TDVF.

Fig. 8.   Second round fitting result of Circuit 2 using TDVF.

By the passivity test we presented earlier, the constant term d in the rational function must be positive. But both fitting methods might end up with a negative d; in which case a simple correction is just to flip the sign of d as long as the rms-error does not change too much, and continue to test the other conditions for passivity. Shown in Table IV are the passivity test results. The two cases in the table indicated "No" were caused by the negative d's. Both circuits passed the passivity test after flipping the sign of d. The rms-errors of both circuits after flipping the sign of d became $9.58E{-}2$ and $13.1E{-}2$, respectively; hence were considered acceptable. In case flipping the sign of d results in large rms-error or still fails the other passivity conditions, then it is recommended to change the fitting orders and test the passivity again.

VI. CONCLUSION

We have investigated a new idea on explicitly extracting delay from rational fitting in both the frequency-domain and the time-domain by using the system response data. We have demonstrated that appreciable delays in inductance-prominent circuits can be identified from the non-minimum phase zeros resulting from rational fitting. A two-round rational fitting procedure is proposed for constructing hybrid interconnect models A quick passivity test in collaborating with fitting is also introduced. Future work includes incorporating the passivity test in the process of fitting and extending the current technique to the hybrid modeling of multiple-input-multiple-output interconnect systems.

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